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Signal and noise separation in prestack seismic data using velocity-dependent seislet transform

Yang Liu[1], Sergey Fomel[2], and Cai Liu[1]

ABSTRACT

The seislet transform is a waveletlike transform that analyzes seismic data by following varying slopes of seismic events across different scales and provides a multiscale orthogonal basis for seismic data. It generalizes the discrete wavelet transform (DWT) in the sense that the DWT in the lateral direction is simply the seislet transform with a zero slope. Our earlier work used plane-wave destruction (PWD) to estimate smoothly varying slopes. However, the PWD operator can be sensitive to strong noise interference, which makes the seislet transform based on PWD (PWD-seislet transform) occasionally fail in providing a sparse multiscale representation for seismic field data. We adopted a new velocity-dependent (VD) formulation of the seislet transform, in which the normal moveout equation served as a bridge between local slope patterns and conventional moveout parameters in the common-midpoint domain. The VD slope has better resistance to strong random noise, which indicated the potential of VD seislets for random noise attenuation under 1D earth assumption. Different slope patterns for primaries and multiples further enabled a VD-seislet frame to separate primaries from multiples when the velocity models of primaries and multiples were well disjoint. We evaluated the results by applying the method to synthetic and field-data examples in which the VD-seislet transform helped in eliminating strong random noise. We performed synthetic and field-data tests that showed the effectiveness of the VD-seislet frame for separation of primaries and peg-leg multiples of different orders.

INTRODUCTION

Signal and noise separation is a persistent problem in seismic exploration. Sometimes, noise is divided into random noise and coherent noise. Many authors have developed effective methods of eliminating random noise. Ristau and Moon (2001) compare several adaptive filters, which they applied in an attempt to reduce random noise in geophysical data. Karsli et al. (2006) apply complex-trace analysis to seismic data for random-noise suppression, recommending it for lowfold seismic data. Some transform methods were also used to deal with seismic random noise, e.g., the discrete cosine transform (Lu and Liu, 2007), the curvelet transform (Neelamani et al., 2008), and the seislet transform (Fomel and Liu, 2010). If seismic events are planar (lines in 2D data and planes in 3D data) or locally planar, one can predict seismic events by using prediction techniques in the f-x domain (Canales, 1984; Sacchi and Kuehl, 2001; Liu and Liu, 2013) or the t-x domain (Claerbout, 1992; Fomel, 2002; Sacchi and Naghizadeh, 2009; Liu et al., 2015).

Multiple reflections are one kind of coherent noise, especially in marine environments. Wave-equation-based algorithms for attenuating multiples have rapidly developed since the 1990s and usually consist of two steps, namely, multiple prediction (Verschuur et al., 1992; Berkhout and Verschuur, 1997; Weglein et al., 1997) and adaptive subtraction (Wang, 2003b; Guitton and Verschuur, 2004; Fomel, 2009a). However, these algorithms need to calculate a full wavefield, which is often a computational bottleneck for their application, especially in the 3D case. Another popular class of demultiple techniques is based on variants of the Radon transform (Foster and Mosher, 1992). Several revised Radon transforms have been proposed for multiple attenuation (Hunt et al., 1996; Zhou and Greenhalgh, 1996; Hargreave et al., 2003; Wang, 2003a). Radon-transform-based methods often fail to provide accurate separation because of their nonsparsity in characterizing seismic data, although they can be improved by high-resolution methods (Sacchi and Ulrych, 1995; Herrmann et al., 2000; Trad et al., 2003). Despite their

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[1]Jilin University, College of Geo-exploration Science and Technology, Changchun, China. E-mail: yangliu1979@jlu.edu.cn; liucai@jlu.edu.cn.
[2]The University of Texas at Austin, Bureau of Economic Geology, John A. and Katherine G. Jackson School of Geosciences, University Station, Austin, Texas, USA. E-mail: sergey.fomel@beg.utexas.edu.

usual classification as noise, multiples can penetrate deeply enough into the subsurface to illuminate the prospect zone. In this sense, multiples can also be viewed as a viable signal, rather than noise (Reiter et al., 1991; Youn and Zhou, 2001; Berkhout and Verschuur, 2006). Brown and Guitton (2005) propose a least-squares joint imaging of peg-leg multiples and primaries and discussed separation of peg-leg multiples and primaries in prestack data.

In seismic data analysis, it is common to represent signals as sums of plane waves by using multidimensional Fourier transforms. The discrete wavelet transform (DWT) is often preferred to the Fourier transform for characterizing digital images because of its ability to localize events in time and frequency domains (Jensen and la Cour-Harbo, 2001; Mallat, 2009). However, DWT may not be optimal for describing data that consist of plane waves. Waveletlike transforms that explore directional characteristics of images have found important applications in seismic imaging and data analysis (Chauris and Nguyen, 2008; Herrmann et al., 2008). Fomel (2006) investigate the possibility of designing a waveletlike transform tailored specifically to seismic data and introduced it as the seislet transform. Fomel and Liu (2010) further develop the seislet framework and propose additional applications. The original 2D seislet transform uses local data slopes estimated by plane-wave destruction (PWD) filters (Fomel, 2002; Chen et al., 2013a, 2013b). However, a PWD operator can be sensitive to strong interference, which makes the seislet transform based on PWD (PWD-seislet transform) occasionally fail in characterizing noisy signals.

We develop a velocity-dependent (VD) concept (Liu and Liu, 2013), where local slopes in prestack data are evaluated from moveout parameters estimated by conventional velocity-analysis techniques. We implement a VD-seislet transform and propose its application for signal and noise separation. We expect the new VD-seislet transform to provide better compression ability for reflection events away from interference of strong random noise. We also provide an application of VD-seislet transform for separating primaries from peg-leg multiples of different orders. We test the performance of VD-seislet transform using synthetic and field data.

THEORY

Review of seislets

The seislet transform was introduced by Fomel (2006) and extended by Fomel and Liu (2010) and Liu and Fomel (2010). The seislet construction is based on the DWT combined with seismic data patterns, such as local slopes or frequencies. Fomel (2002) develops a local PWD operation to predict local plane-wave events, in which an all-pass digital filter is used to approximate the time shift between two neighboring traces. The inverse operation, plane-wave construction (Fomel and Guitton, 2006; Fomel, 2010), predicts a seismic trace from its neighbors by following locally varying slopes of seismic events and has been used for designing a PWD-seislet transform, which is a particular kind of the seislet transforms with slope pattern. Liu and Liu (2013) propose a VD slope as a pattern in VD-seislet transform, in which the normal-moveout (NMO) equation serves as a bridge between local slopes and scanned NMO velocities.

To define seislet transform, we follow the general recipe of the lifting scheme for the DWT, as described by Sweldens and Schröder (1996). The construction is reviewed in Appendix A. Designing pattern-based prediction operator $\mathbf{P}$ and update operator $\mathbf{U}$ for seismic data is a key factor in the seislet framework. In the seislet transform, the basic data components can be different, e.g., traces or common-offset gathers, and the prediction and update operators shift components according to different patterns.

The prediction and update operators for a simple seislet transform are defined by modifying the biorthogonal wavelet construction in equations from Appendix A as follows:

$$\mathbf{P}[\mathbf{e}]_k = (\mathbf{R}_k^{(+)}[\mathbf{e}_{k-1}] + \mathbf{R}_k^{(-)}[\mathbf{e}_k])/2 \tag{1}$$

and

$$\mathbf{U}[\mathbf{r}]_k = (\mathbf{R}_k^{(+)}[\mathbf{r}_{k-1}] + \mathbf{R}_k^{(-)}[\mathbf{r}_k])/4, \tag{2}$$

where $\mathbf{e}_k$ is even components of data at the kth transform scale, $\mathbf{r}_k$ is residual difference between the odd component of data $\mathbf{o}$ and its prediction from the even component at the kth transform scale, the details are shown in Appendix A, and $\mathbf{R}_k^{(+)}$ and $\mathbf{R}_k^{(-)}$ are operators that predict a component from its left and right neighbors correspondingly by shifting them according to their patterns.

To get the relationship between prediction operator $\mathbf{R}_k$ and slope pattern σ, the PWD operation (Fomel, 2002) can be defined in a linear operator notation as

$$\mathbf{d} = \mathbf{D}(\sigma)\mathbf{s}, \tag{3}$$

where seismic section $\mathbf{s} = [\mathbf{s}_1 \quad \mathbf{s}_2 \quad , \ldots, \quad \mathbf{s}_N]^T$ is a collection of traces, $\mathbf{d}$ is the destruction residual. The general structure of $\mathbf{D}$ is defined as follows (Fomel and Guitton, 2006; Fomel, 2010):

$$\mathbf{D}(\boldsymbol{\sigma}) = \begin{bmatrix} \mathbf{I} & 0 & 0 & \cdots & 0 \\ -\mathbf{R}_{1,2}(\sigma_1) & \mathbf{I} & 0 & \cdots & 0 \\ 0 & -\mathbf{R}_{2,3}(\sigma_2) & \mathbf{I} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -\mathbf{R}_{N-1,N}(\sigma_{N-1}) & \mathbf{I} \end{bmatrix}, \tag{4}$$

where $\mathbf{I}$ stands for the identity operator, σ_i is the local slope pattern, and $\mathbf{R}_{i,j}(\sigma_i)$ is an operator for prediction of trace j from trace i according to the slope pattern σ_i. A trace is predicted by shifting it according to the local seismic event slopes. Prediction of a trace from a distant neighbor can be accomplished by simple recursion, i.e., predicting trace k from trace 1 is simply given as

$$\mathbf{R}_{1,k} = \mathbf{R}_{k-1,k}, \ \ldots, \mathbf{R}_{2,3}\mathbf{R}_{1,2}. \tag{5}$$

If $\mathbf{s}_r$ is a reference trace, then the prediction of trace $\mathbf{s}_k$ is $\mathbf{R}_{r,k}\mathbf{s}_r$.

The predictions need to operate at different scales, which, in this case, mean different separation distances between the data elements, e.g., traces in PWD-seislet transform. Equations 1 and 2, in combination with the forward and inverse lifting schemes, provide a complete definition of the seislet framework. For different kinds of slope-based seislets, one needs to define the corresponding slope pattern σ.

Velocity-dependent-slope pattern for primary reflections

The kinematic description of a seismic event is an essential step for several developments in seismic data processing. Local slope is one important kinematic pattern for seismic data in the time-space domain. PWD provides a constructive algorithm for estimating local slopes (Claerbout, 1992; Fomel, 2002; Schleicher et al., 2009; Chen et al., 2013a, 2013b) and can combine with a seislet framework to implement the PWD seislet. The local slant stack (Ottolini, 1983a) is another standard tool for calculating slopes.

Under a 1D earth assumption, one can consider the classic hyperbolic model of primary reflection moveouts at near offsets (Dix, 1955):

$$t(x) = \sqrt{t_0^2 + \frac{x^2}{v^2(t_0)}}, \tag{6}$$

where t_0 is the zero-offset traveltime, $t(x)$ is the corresponding primary traveltime recorded at offset x, and $v(t_0)$ is the stacking, or root-mean-square (rms) velocity, which comes from a standard velocity scan. As follows from equation 6, the traveltime slopes $\sigma = dt/dx$ in common-midpoint (CMP) gathers are given by

$$\sigma(t,x) = \frac{x}{t(x)v^2(t_0,x)}. \tag{7}$$

This calculation is reverse to the one used in NMO by velocity-independent imaging (Ottolini, 1983b; Fomel, 2007b). To calculate local slopes of primaries, we need to know $v(t_0,x)$ at each time-space location (t_0,x). This can be accomplished by simultaneously scanning t_0 and $v(t_0,x)$ according to the hyperbolic NMO equation at each x-coordinate position or by time warping. We use the time warping algorithm to calculate $v(t_0,x)$; the time-warping performs mapping between different coordinates, if one has sampled functions $f(x)$ and $y(x)$, the mapping operation finds sampled $f(y)$ (Burnett and Fomel, 2009; Casasanta and Fomel, 2011).

After the VD-slope pattern of primaries is calculated, we can design pattern-based prediction and update operators $\mathbf{R}_k$ by using plane-wave construction for the VD-seislet transform to represent only primary reflections. When VD-seislet transform is applied to a CMP gather, random noise spreads over different scales whereas the predictable reflection information gets compressed to large coefficients at small scales. A simple thresholding operation can easily remove small coefficients. Finally, applying the inverse VD-seislet transform reconstructs the signal while attenuating random noise.

Velocity-dependent-slope pattern for peg-leg multiples

In a laterally homogeneous model, the NMO equation 6 flattens primary events on a CMP gather with offset x and time t to its zero-offset traveltime t_0. Brown and Guitton (2005) use an analogous NMO equation for peg-leg multiples under a locally 1D earth assumption. For example, a first-order peg-leg can be kinematically approximated by a pseudoprimary with the same offset but with an additional zero-offset traveltime τ. The NMO equation for an mth-order peg-leg multiple is generalized to

$$t_m(x) = \sqrt{(t_0 + m\tau)^2 + \frac{x^2}{v_m^2(t_0)}}, \tag{8}$$

where $t_m(x)$ is the corresponding multiple traveltime recorded at offset x and the effective rms velocity v_m is defined according to Dix's equation as

$$v_m(t_0) = \sqrt{\frac{t_0 v^2(t_0) + m\tau v^2(\tau)}{t_0 + m\tau}}. \tag{9}$$

In marine seismic data, $v(\tau)$ is a constant water velocity, and it assumes that we are able to pick zero-offset traveltime τ of the water bottom. According to the definition of slopes for primaries (equation 7), slopes for peg-leg multiples can be calculated analogously by

$$\sigma_m(t,x) = \frac{x}{t_m(x)v_m^2(t_0,x)}. \tag{10}$$

Equation 10 provides the estimation of multiple slopes, which we use to define VD-seislet frame for representing peg-leg multiples of different orders.

Separation of primaries and peg-leg multiples using Velocity-dependent-seislet frame

Once the VD-seislet transform is defined, it can be applied to analyze signals composed of multiple wavefields, e.g., primaries and multiples of different orders. If a range of slopes are chosen and a VD-seislet transform is constructed for each of them, then all the transforms together will constitute an overcomplete representation. Mathematically, if $\mathbf{F}_i$ is the VD-seislet transform for the ith slope pattern (corresponding to primaries or peg-leg multiples of different orders), then, for any data vector $\mathbf{d}$,

$$\sum_{i=1}^{N}\|\mathbf{F}_i\mathbf{d}\|^2 = \sum_{i=1}^{N}\mathbf{d}^T\mathbf{F}_i^T\mathbf{F}_i\mathbf{d} = \sum_{i=1}^{N}\|\mathbf{d}\|^2 = N\|\mathbf{d}\|^2, \tag{11}$$

which means that all transforms taken together constitute a tight frame with constant N (Mallat, 2009).

Because of its overcompleteness, a frame representation for a given signal is not unique. To assure that different wavefield components do not leak into other parts of the frame, it is advantageous to use a sparsity-promoting inversion (Fomel and Liu, 2010). We use a nonlinear shaping-regularization scheme (Fomel, 2008) and define sparse decomposition as an iterative thresholding process (Daubechies et al., 2004) as

$$\hat{\mathbf{f}}_{k+1} = \mathbf{S}[\mathbf{Fd} + (\mathbf{I} - \mathbf{FB})\hat{\mathbf{f}}_k] \tag{12}$$

and

$$\mathbf{f}_{k+1} = \mathbf{f}_k + \mathbf{Fd} - \mathbf{FB}\hat{\mathbf{f}}_{k+1}, \tag{13}$$

where $\mathbf{f}_k$ are coefficients of the seislet frame at kth iteration, $\hat{\mathbf{f}}_k$ is an auxiliary quantity, $\mathbf{S}$ is a soft thresholding operator, $\mathbf{F}$ and $\mathbf{B}$ are the frame construction and deconstruction operators:

$$\mathbf{F} \equiv [\mathbf{F}_1 \quad \mathbf{F}_2 \quad , \ldots , \quad \mathbf{F}_N]^T \tag{14}$$

and

$$\mathbf{B} \equiv [\,\mathbf{F}_1^{-1} \quad \mathbf{F}_2^{-1} \quad , \ldots , \quad \mathbf{F}_N^{-1}\,]. \tag{15}$$

Equation 12 converges to the solution of a least-squares optimization problem regularized by a sparsity constraint:

$$\min_{\mathbf{f}} \|\mathbf{Bf} - \mathbf{d}\|_2^2 + \epsilon \|\mathbf{f}\|_1, \tag{16}$$

where the first term is the L_2 norm of the data misfit, and the second term is the L_1 norm of the model, which promotes sparsity.

Assuming that equation 12 converges, one can express its convergence point as

$$\hat{\mathbf{f}} = [\mathbf{I} + \mathbf{S}(\mathbf{FB} - \mathbf{I})]^{-1}\mathbf{SFd}, \tag{17}$$

which is precisely the shaping regularization equation proposed by Fomel (2007a). Iterations in equations 12 and 13 start with $\mathbf{f}_0 = \mathbf{0}$ and $\hat{\mathbf{f}_0} = \mathbf{Fd}$ and are related to the linearized Bregman iteration (Cai et al., 2009), which converges to the solution of the constrained minimization problem:

$$\min_{\mathbf{f}} \|\mathbf{f}\|_1 \text{ s.t. } \mathbf{Bf} = \mathbf{d}. \tag{18}$$

Separated wavefield can be calculated by $\mathbf{d}_i = \mathbf{BM}_i\mathbf{f}_\eta$, where η is iteration number, and masking operator $\mathbf{M}_i$ is a diagonal matrix that is given as

$$\mathbf{M}_i = \begin{bmatrix} \mathbf{0} & \cdots & \cdots & \cdots & \cdots & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{I}_{i,i} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \cdots & \cdots & \cdots & \cdots & \mathbf{0} \end{bmatrix}_{N\times N}, \tag{19}$$

and $\mathbf{d}_i$ corresponds to the signal of interest (e.g., primaries or multiples of a selected order). One needs to calculate all patterns for primaries and multiples, and then sparse decomposition (equations 12 and 13) will separate primaries from multiples. In practice, a small number of iterations is usually sufficient for convergence and for achieving model sparseness and data recovery. However, similar to Radon transform, the proposed method needs an assumption that primaries and multiples correspond to different velocity models.

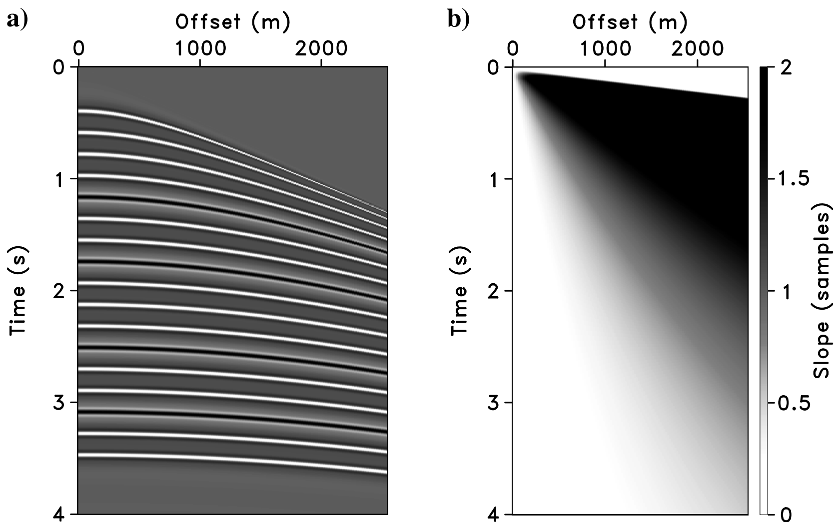

Figure 1. (a) Synthetic data and (b) slopes calculated by PWD.

SYNTHETIC DATA EXAMPLES

Validation of slope estimation and random noise elimination

A simple synthetic example is shown in Figure 1a. The synthetic data were generated by applying inverse NMO with time-varying velocities, and they represent perfectly hyperbolic events. Figure 1b shows local event slopes measured from the data using PWD algorithm (Fomel, 2002). PWD provides an accurate slope field for noise-free data. Figure 2a and 2b shows the data after adding normally distributed random noise and local slopes from PWD, respectively. Compared

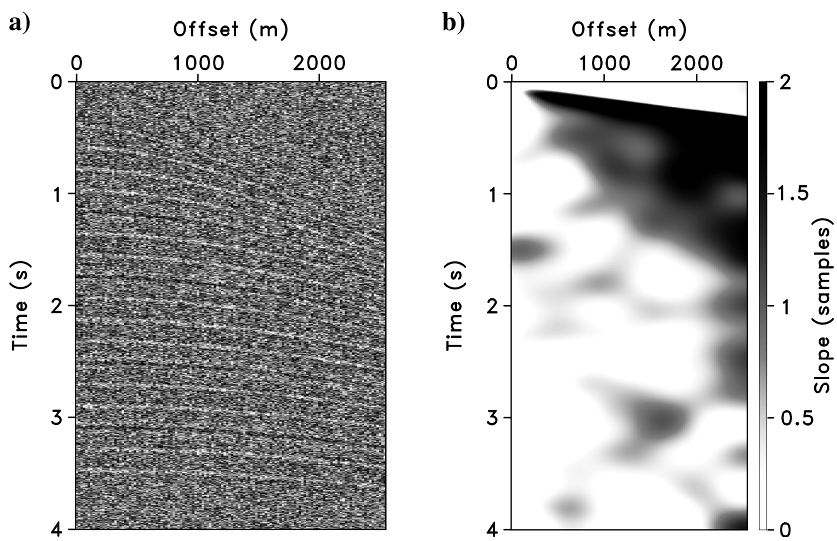

Figure 2. (a) Synthetic noisy data and (b) slopes calculated by PWD.

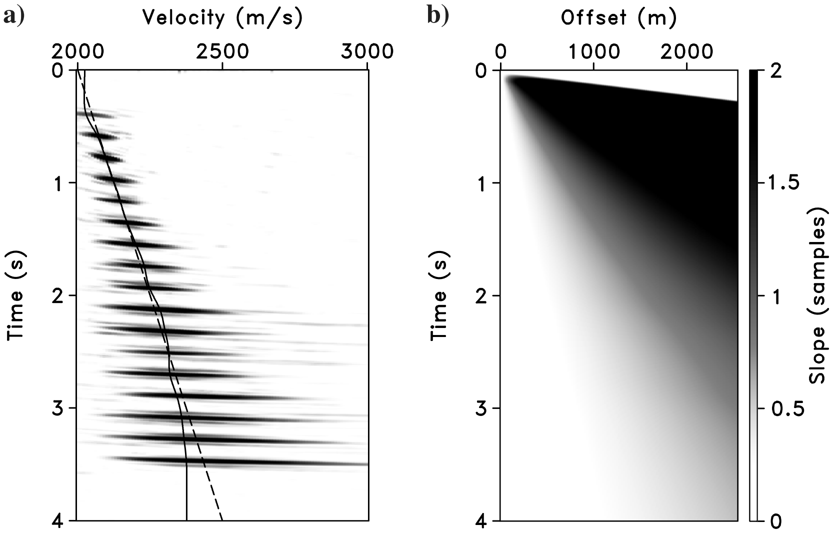

Figure 3. (a) Velocity scanning (dashed line: exact velocity, solid line: picked velocity) and (b) VD slopes.

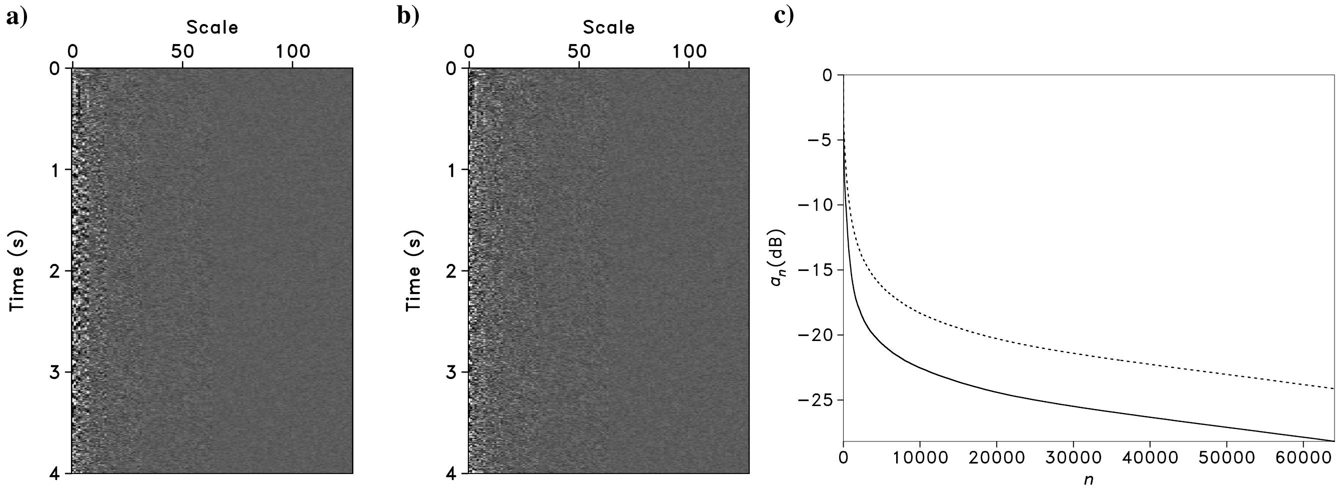

Figure 4. (a) PWD-seislet coefficients, (b) VD-seislet coefficients, and (c) transform coefficients sorted from large to small, normalized, and plotted on a decibel scale (solid line — VD-seislet transform. Dashed line — PWD-seislet transform).

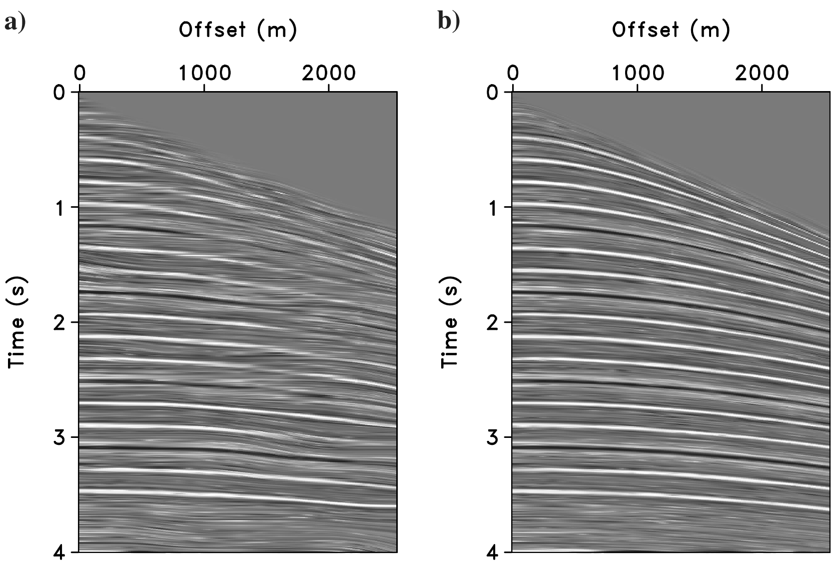

Figure 5. Denoising result using different transforms. (a) PWD-seislet transform and (b) VD-seislet transform.

with Figure 1b, PWD fails in finding exact slope field because of strong random noise. Next, we calculate slopes using NMO velocities from velocity scan. Picked NMO velocities (Figure 3a) are close to the exact velocity because velocity scan is less sensitive to strong random noise. As a consequence, VD slopes calculated from equation 7 provide a more accurate result (Figure 3b).

A direct application of the seislet transform is denoising. We apply PWD-seislet and VD-seislet transforms on the noisy data (Figure 2a). Figure 4a and 4b shows the transform coefficients of PWD-seislet and VD-seislet, respectively. The hyperbolic events are compressed in both of the transform domains. Notice that the PWD-seislet coefficients get more concentrated at a small scale than those of the VD-seislet because part of the random noise is also compressed along the inaccurate PWD slopes. Meanwhile, random noise gets spread over different scales in the VD-seislet domain, whereas the predictable reflection information gets compressed to large coefficients at small scales, which makes signal and noise display different amplitude characteristics. Figure 4c shows a comparison between the decay of coefficients sorted from large to small in the PWD-seislet transform and the VD-seislet transform. Seislet transform can compress the seismic events with coincident wavelets; if the slopes of the reflections are correct, the sparse large coefficients only correspond to the stacked reflection events. However, when the slopes of the reflections are not accurate, the stacked amplitude values for inconsistent wavelets will create more coefficients with smaller values. VD slopes are less affected by strong random noise than are PWD slopes; therefore, they show a faster decay of the VD-seislet coefficients. A simple thresholding method can easily remove the small coefficients of random noise. Figure 5a and 5b displays the denoising results by using PWD-seislet transform and VD-seislet transform, respectively. The events after PWD-seislet transform denoising show serious distortion whereas VD-seislet transform produces a reasonable denoising result. For numerical comparison, we use the signal-to-noise ratio (S/N) defined as $S/N = 10\log_{10}\|\mathbf{s}\|_2^2/\|\mathbf{s}-\hat{\mathbf{s}}\|_2^2$, where $\mathbf{s}$ is the noise-free signal and $\hat{\mathbf{s}}$ is the denoised signal. The original S/N of the noisy data (Figure 2a) is −12.53 dB. The S/N of the denoised results using the PWD-seislet transform (Figure 5a) and the VD-seislet transform (Figure 5b) are 0.53 and 1.94 dB, respectively.

Separation of primaries and peg-leg multiples

Next, we use a synthetic CMP gather (Figure 6a) to test the separation of primaries and peg-leg multiples by VD-seislet frame. This gather was generated by Lumley et al. (1994) using Haskell-Thompson elastic modeling and a well log from the Mobil amplitude-variation-with-offset (AVO) data set (Keys and Foster, 1998). The gather contains primaries and water-bottom multiples of different orders.

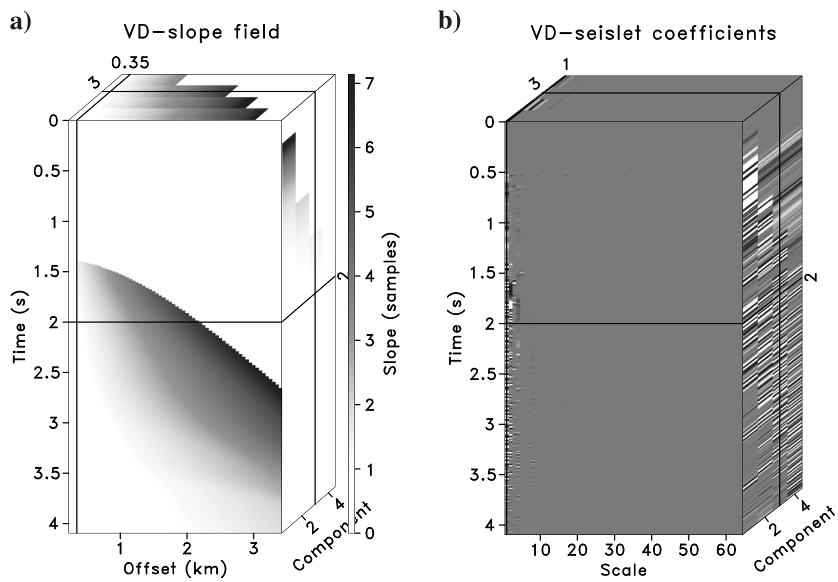

Figure 7. (a) VD slopes and (b) VD-seislet coefficients.

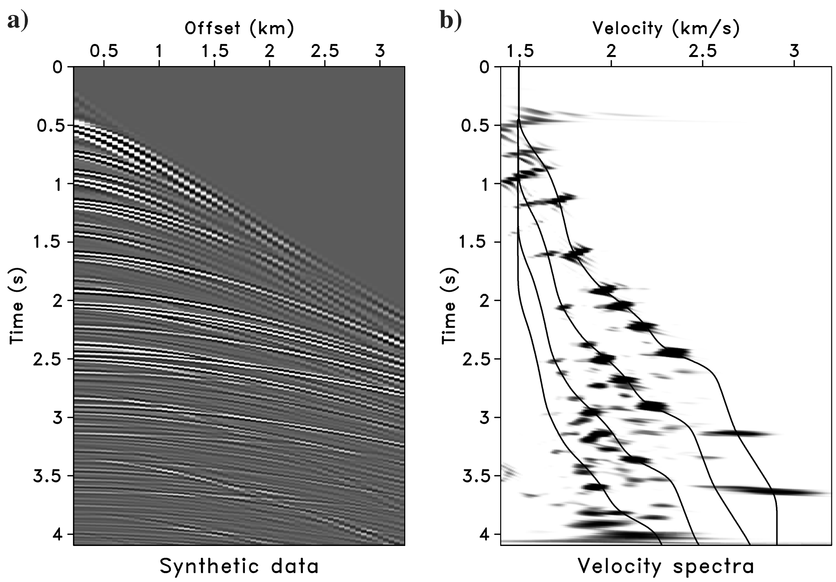

Figure 6. (a) Synthetic model and (b) velocity trends of primaries and multiples.

To separate peg-leg multiples from primaries, we transform the data using VD-seislet frame by involving different VD slope fields (Figure 7a) according to equations 7 and 10. The primary velocities and calculated velocities of different-order peg-leg multiples are shown in Figure 6b. The estimated curves of multiple velocities indicate accurate trends in the velocity spectra. The proposed method uses a nonlinear shaping-regularization scheme (equations 12 and 13) to separate different wavefield components, which are shaped to be sparse in the corresponding VD-seislet frame domain (Figure 7b). In this example, the pattern number N is selected to be four in equation 11. The separated wavefields are shown in Figure 8 and display reasonably accurate separation results.

FIELD DATA EXAMPLES

We test VD-seislet denoising by using a field-land data provided by Geofizyka Torun Sp. Z.o.o, Poland from FreeUSP (2015) website. Figure 9a shows the CMP gathers after removing most of the ground roll. The strong random noise makes reflection events hardly visible. However, velocity analysis from equation 6 can still produce a reasonable velocity field. Equation 7 converts rms velocity to seismic pattern (Figure 9b), which displays the hyperbolic slopes from negative to positive in CMP gathers and varying slopes in common-offset section. VD-seislet transform uses the slopes to compress reflections along offset axis. Figure 9c shows the VD-seislet coefficients, in which the small dynamic range of seislet coefficients implies a good compression ratio. If we choose the significant coefficients at the coarse scale, e.g., scale < 8, and zero-out difference coefficients at the finer scales, the inverse transform effectively removes incoherent noise from the gathers (Figure 9d).

Next, we test the proposed algorithm to separate multiwavefields on a single CMP gather from the Viking Graben (Mobil AVO) data set (Keys and Foster, 1998). The field data are shown in Figure 10a.

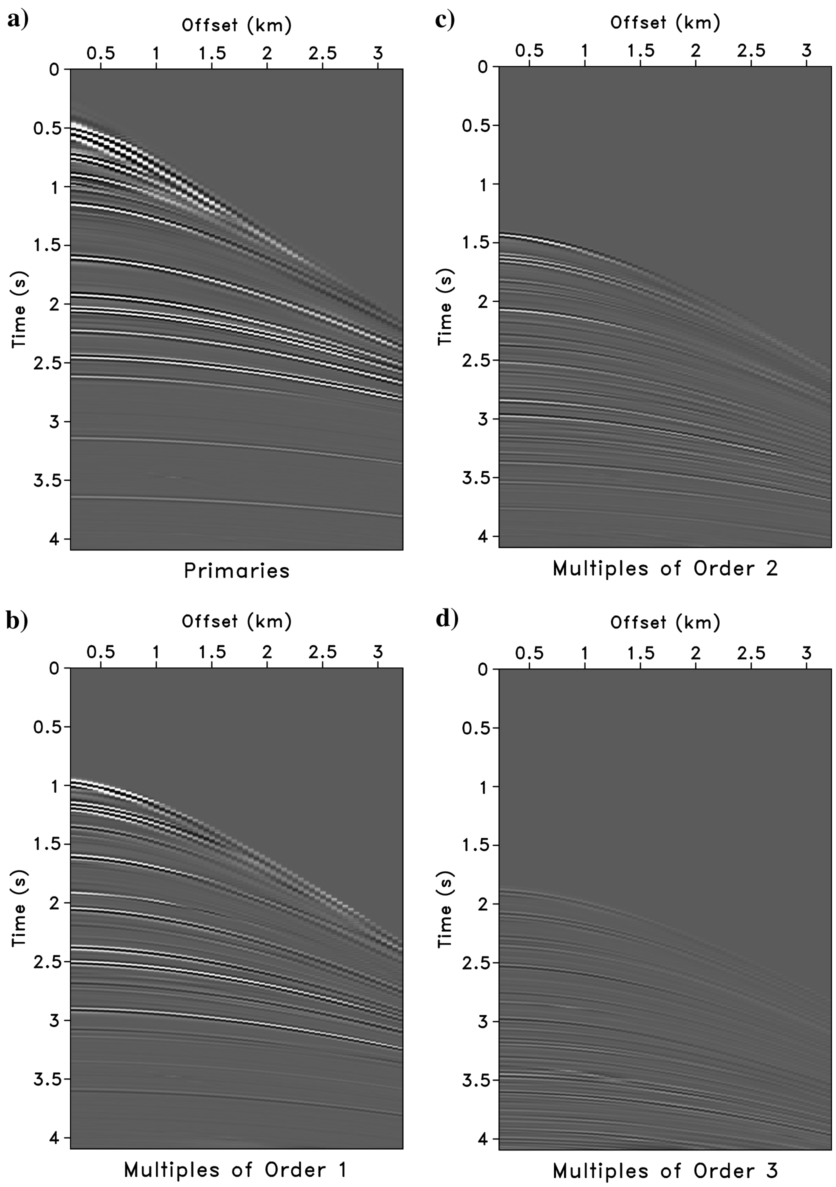

Figure 8. Separated wavefields. (a) Primaries, (b) first-order multiples, (c) second-order multiples, and (d) third-order multiples.

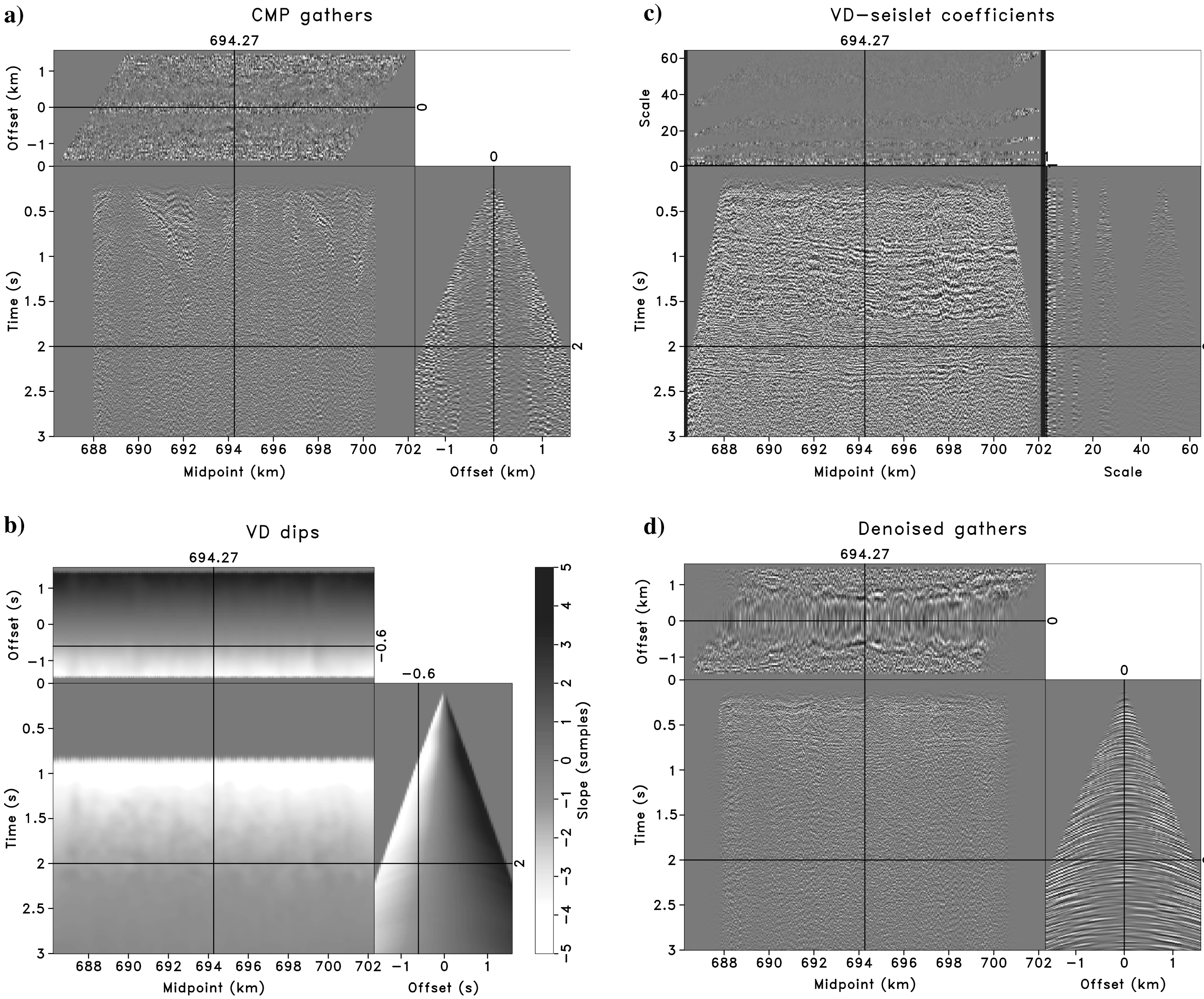

Figure 9. (a) Field CMP gathers, (b) VD slopes, (c) VD-seislet coefficients, and (d) denoising result using VD-seislet transform.

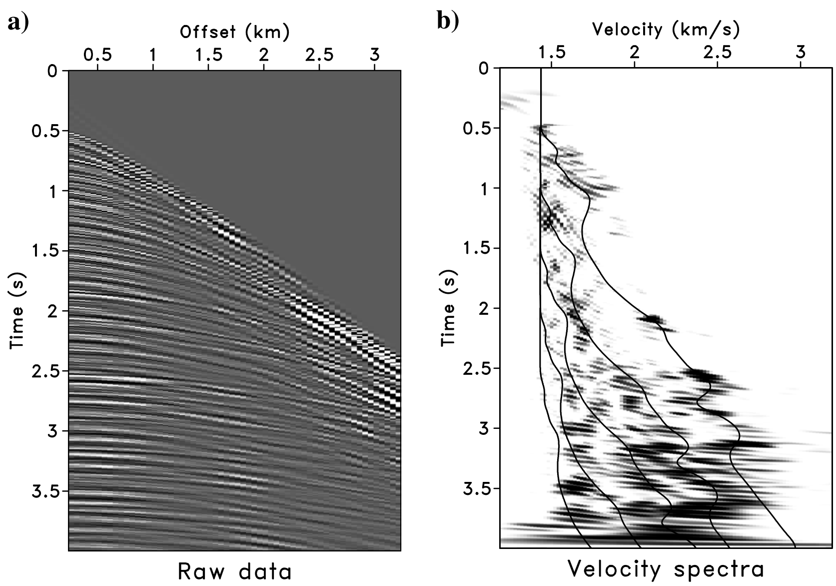

Figure 10. (a) Field CMP gather and (b) velocity trends of primaries and multiples.

We pick the primary velocities by muting spectra energy of multiples. Multiple rms velocities with different orders (equation 9) follow the pseudoprimary NMO equation 8. The velocity spectra of primaries and multiples are shown in Figure 10b. Equations 7 and 10 convert velocities to slopes, which help the VD-seislet frame separate primaries from different-order peg-leg multiples. (We only display three orders.) Figure 11 displays the separated primaries and different-order multiples. The corresponding velocity spectra are shown in Figure 12. After separating different wavefields, the velocity spectra confirm that the signals get concentrated around their respective trends. All results are reproducible in the Madagascar open-source software environment (Fomel et al., 2013).

DISCUSSION

What are the limitations of the proposed algorithms? First, the VD-seislet transform may have difficulties in dealing with nonhyperbolic moveouts. However, it is possible to extend it to the nonhyperbolic moveout equation, which would provide the possibility to handle large offsets and anisotropy (Fomel and Grechka, 2001). Next, the velocity scan occasionally raises additional errors; it is possible to predenoise followed by PWD-seislet transform, but the results depend on careful parameter selection for preprocessing methods. Compared with PWD-seislet transform, VD-seislet transform provides more accurate representation for seismic events with class II AVO anomalies (Rutherford and Williams, 1989) that cause seismic amplitudes to go through a polarity reversal. The PWD-seislet transform may fail because PWD slopes depend on the amplitude value of the seismic data; however, VD slopes are more accurate by using advanced velocity analysis, e.g., AB semblance (Fomel, 2009b). Finally, the proposed method works for the velocity models in which primaries and multiples are well disjointed, for instance, it may fail in the case of shallow water multiples. Therefore, the current VD-seislet transform provides an alternative tool for analyzing CMP gathers with strong random noise in structurally simple areas or disjoint peg-leg multiples.

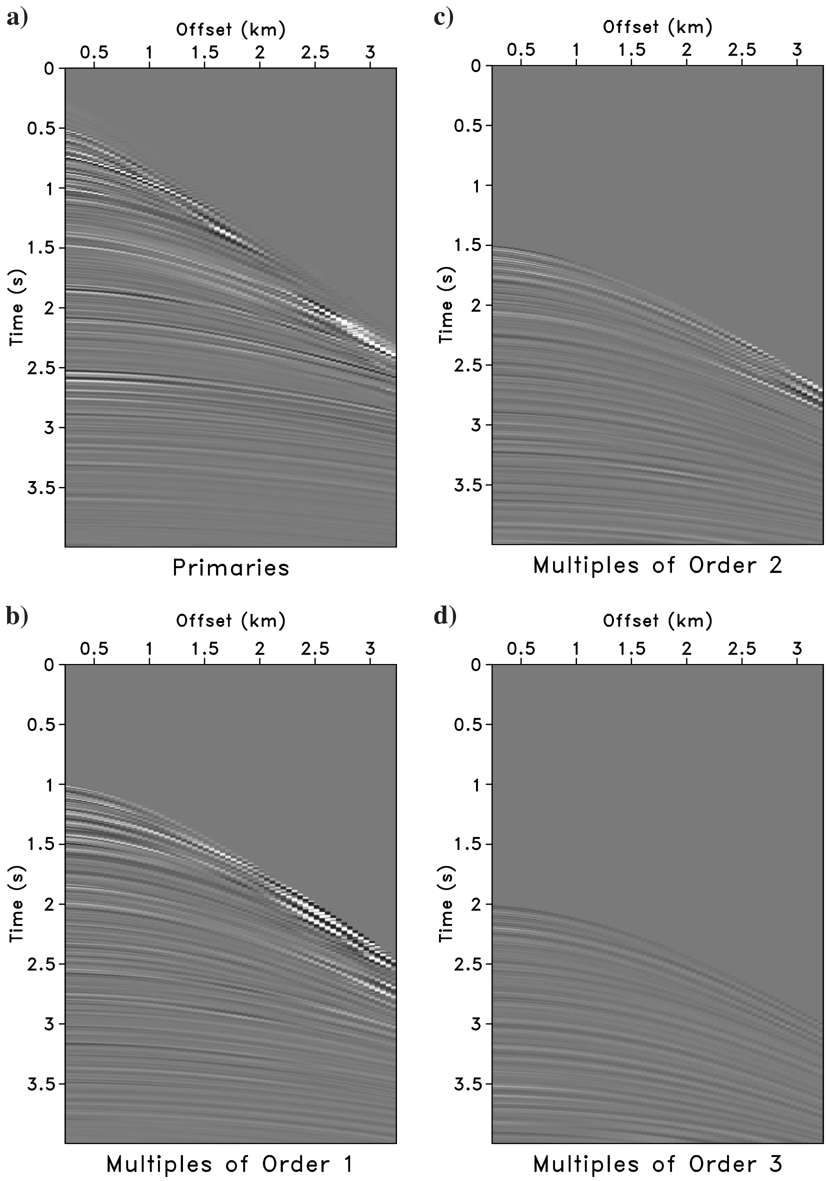

Figure 11. Separated wavefields. (a) Primaries, (b) first-order multiples, (c) second-order multiples, and (d) third-order multiples.

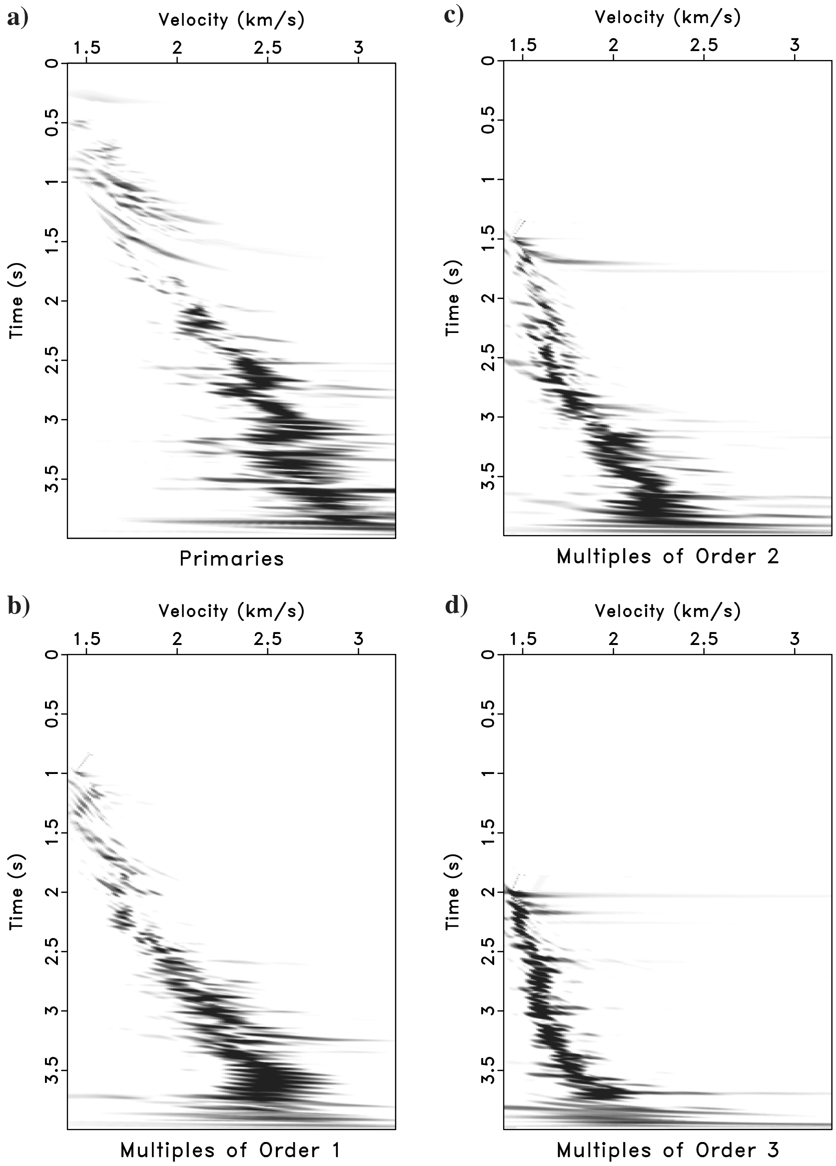

Figure 12. Velocity spectra of different wavefields. (a) Primaries, (b) first-order multiples, (c) second-order multiples, and (d) third-order multiples.

CONCLUSIONS

We have introduced the VD-seislet transform, a new domain for analyzing prestack reflection data in CMP domain. The new transform is able to compress reflection data away from strong random noise. The NMO equation serves as a bridge between local slopes and scanned velocities that are not sensitive to strong random noise and aliasing. We also used the explicit relationship between slopes and velocities of primary and multiple events to extend VD-seislet transform to a VD-seislet frame. We have shown example applications of VD-seislet transform to signal and noise separation. Other traditional processing tasks such as data interpolation can also be easily defined in the VD-seislet domain.

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APPENDIX A

THE LIFTING SCHEME FOR DISCRETE WAVELET TRANSFORM

The lifting scheme (Sweldens, 1995) provides a convenient approach for defining wavelet transforms by breaking them down into the following steps:

1) Divide data into even and odd components $\mathbf{e}$ and $\mathbf{o}$.
2) Find a residual difference $\mathbf{r}$ between the odd component and its prediction from the even component:

$$\mathbf{r} = \mathbf{o} - \mathbf{P}[\mathbf{e}], \tag{A-1}$$

where $\mathbf{P}$ is a prediction operator.

3) Find a coarse approximation **c** of the data by updating the even component as

$$\mathbf{c} = \mathbf{e} + \mathbf{U}[\mathbf{r}], \tag{A-2}$$

where **U** is an update operator.
4) The coarse approximation **c** becomes the new data, and the sequence of steps is repeated at the next scale.

The Cohen-Daubechies-Feauveau (CDF) 5/3 biorthogonal wavelets (Cohen et al., 1992) are constructed by making the prediction operator a linear interpolation between two neighboring samples as

$$\mathbf{P}[\mathbf{e}]_k = (\mathbf{e}_{k-1} + \mathbf{e}_k)/2, \tag{A-3}$$

and by constructing the update operator to preserve the running average of the signal (Cohen et al., 1992) as follows:

$$\mathbf{U}[\mathbf{r}]_k = (\mathbf{r}_{k-1} + \mathbf{r}_k)/4. \tag{A-4}$$

Furthermore, one can create a high-order CDF 9/7 biorthogonal wavelet transform by using CDF 5/3 biorthogonal wavelets twice with different lifting operator coefficients (Lian et al., 2001). The transform is easily inverted according to reversing the steps above as follows:

1) Start with the coarsest scale data representation **c** and the coarsest scale residual **r**.
2) Reconstruct the even component **e** by reversing the operation in equation A-2 as follows:

$$\mathbf{e} = \mathbf{c} - \mathbf{U}[\mathbf{r}]. \tag{A-5}$$

3) Reconstruct the odd component **o** by reversing the operation in equation A-1 as follows:

$$\mathbf{o} = \mathbf{r} + \mathbf{P}[\mathbf{e}]. \tag{A-6}$$

4) Combine the odd and even components to generate the data at the previous scale level and repeat the sequence of steps.

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